Unsupervised Learning

A Belief Propagation Demo

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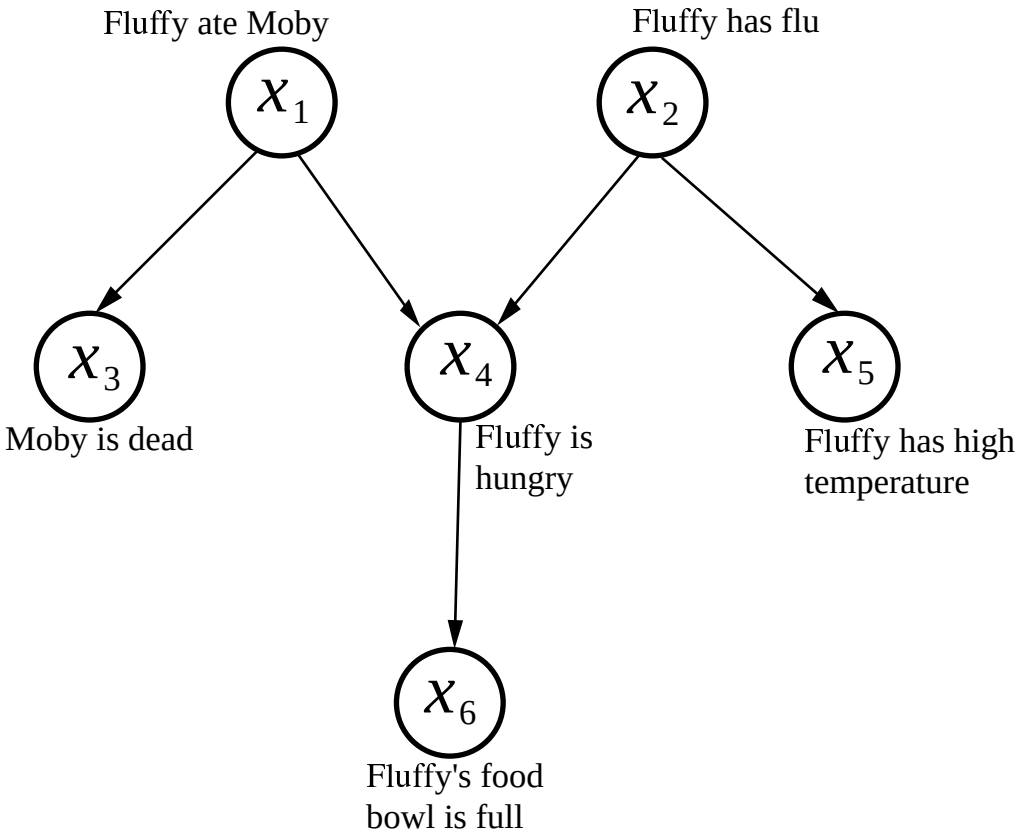
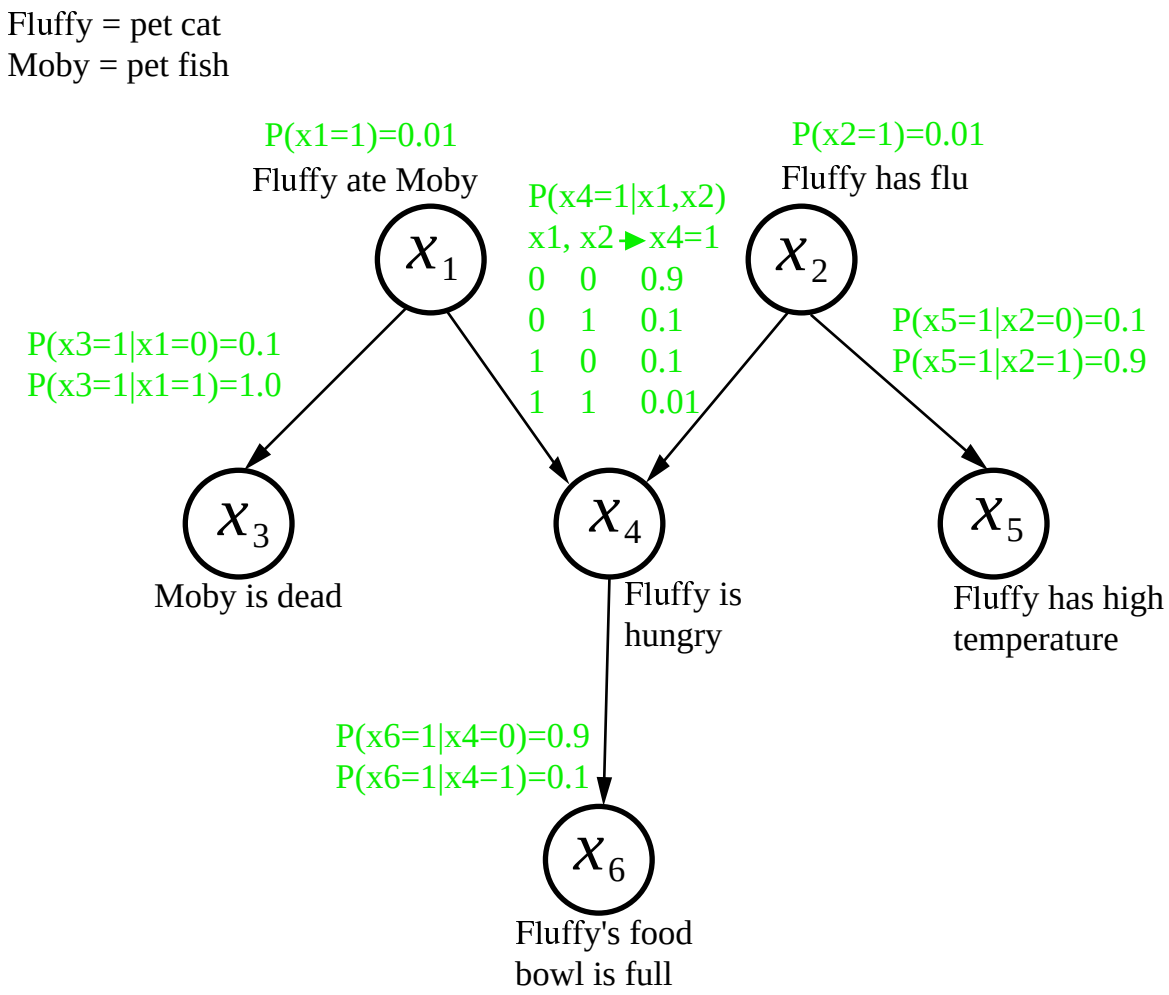
1. Model Structure

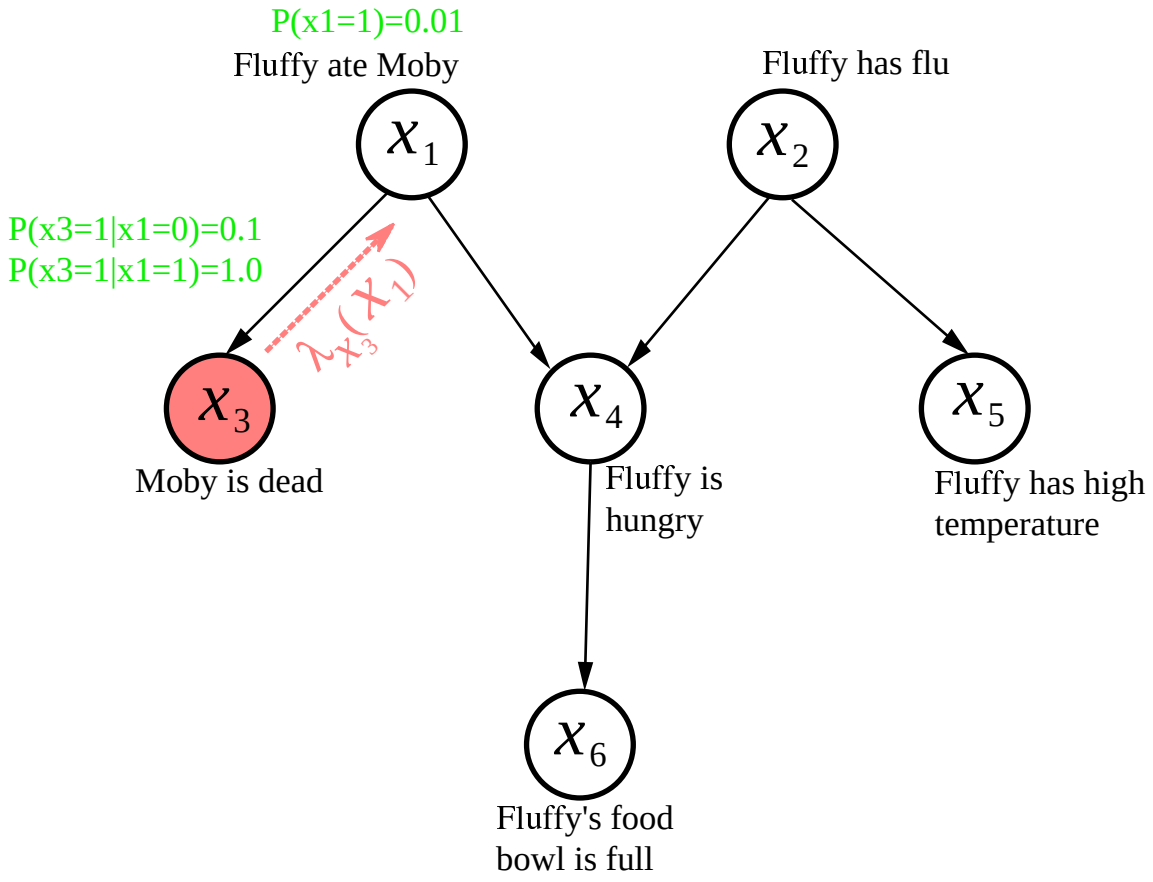
Fluffy = pet cat
Moby = pet fish



2. Model Parameters

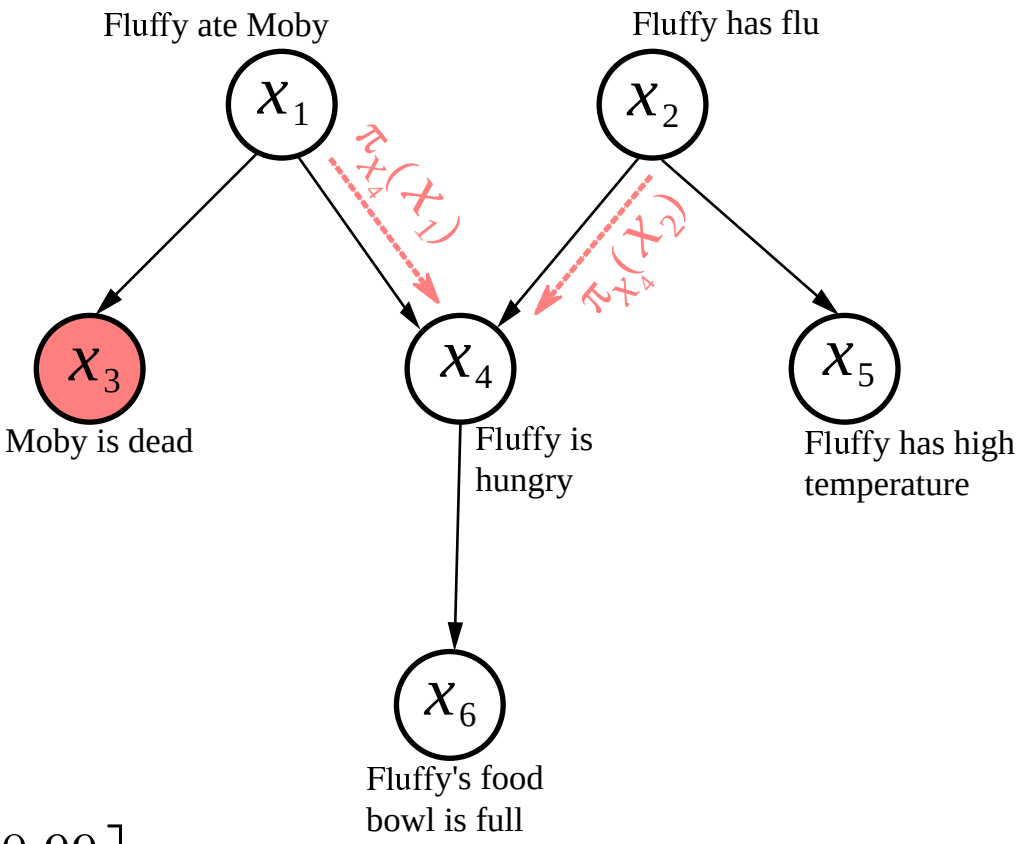


3. Propagating Evidence



1. Observe "Moby is dead", i.e. $x_3 = 1$
2. Send $\lambda_{x_3}(x_1) \equiv p(e^-_{x_1 \to x_3}|x_1) = \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix}$ message $x_3 \to x_1$
3. $BEL(x_1|x_3 = 1) = \frac{1}{Z} \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix} \odot \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.09 \end{bmatrix}$

4. Propagating Evidence



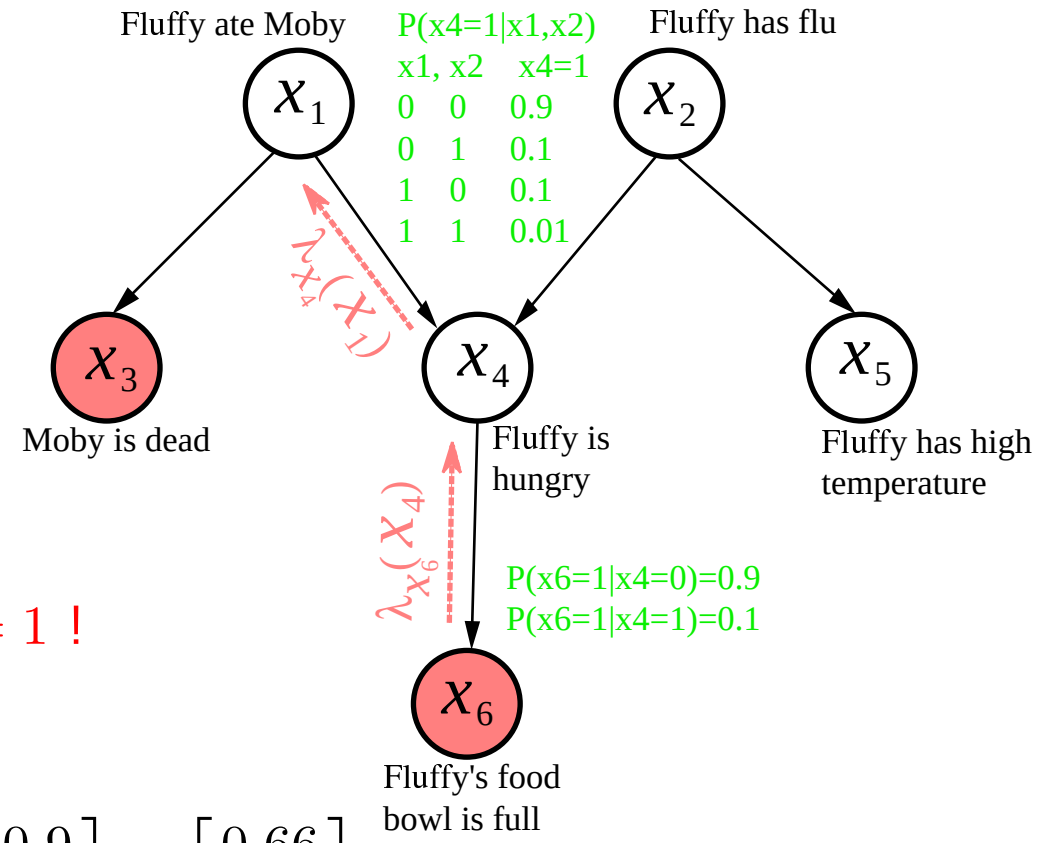
4. Send $\pi_{x_4}(x_1) \equiv p(x_1|e^+_{x_1 \to x_4}) = \begin{bmatrix} 0.91 \\ 0.09 \end{bmatrix}$

5. Send $\pi_{x_4}(x_2) \equiv p(x_2|e^+_{x_2 \to x_4}) = p(x_2) = \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix}$ from $x_2 \to x_4$.

6. Compute $\pi(x_4) \equiv p(x_4|e^+_{x_4}) = \sum_{x_1,x_2} p(x_4|x_1,x_2)\pi_{x_4}(x_1)\pi_{x_4}(x_2) = \begin{bmatrix} 0.18 \\ 0.82 \end{bmatrix}$

7. $BEL(x_4|x_3 = 1) = \begin{bmatrix} 0.18 \\ 0.82 \end{bmatrix}$, whereas before observing $x_3 = 1$, $BEL(x_4) = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$.

5. Propagating Evidence



8. Observe "Fluffy's Food Bowl is Full" $x_6 = 1$!
9. Send $\lambda_{x_6}(x_4) = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$ message $x_6 \rightarrow x_4$
10. $BEL(x_4|x_3 = 1, x_6 = 1) = \frac{1}{Z} \begin{bmatrix} 0.18 \\ 0.82 \end{bmatrix} \odot \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.66 \\ 0.34 \end{bmatrix}$
11. Send $\lambda_{x_4}(x_1) = \sum_{x_4} \lambda_{x_6}(x_4) \sum_{x_2} p(x_4|x_1, x_2)\pi_{x_4}(x_2) = \begin{bmatrix} 0.19 \\ 0.82 \end{bmatrix}$
12. $BEL(x_1|x_3 = 1, x_6 = 1) = \frac{1}{Z} \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix} \odot \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix} \odot \begin{bmatrix} 0.19 \\ 0.82 \end{bmatrix} = \begin{bmatrix} 0.70 \\ 0.30 \end{bmatrix} \Rightarrow$ Fluffy still innocent!